

Kinds of monsters and kinds of compositionality

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1 Introduction

Recently in this journal Stefano Predelli (2014) has argued that there are three different kinds of monsters, each suggested by different remarks David Kaplan makes in ‘Demonstratives’ (1977). This seems to complicate a claim endorsed by some recent writers concerning the compositionality of monsters: that they are character-compositional but not content-compositional (Westerståhl 2012, §6.3, Rabern 2013, 394n.1, Yli-Vakkuri 2013, 265–66, 273n.45). It seems that we should ask the compositionality question separately, concerning each type of monster.

But things are simpler than that, because attention to the compositionality question makes trouble for Predelli’s distinction. For while the first of Predelli’s categories is introduced by means of the *mechanism* it involves – ‘shifting’ of contexts during evaluation – the third category is specified in terms of compositional *behaviour*: monsters in that category ‘operate on character.’ We will find, however, that the examples introduced under the first heading have the very behaviour that is claimed to be distinctive of the third kind. (I will have little to say concerning the second kind of monster that Predelli distinguishes, other than to argue that their very definability is dubious.)

2 Two kinds of compositionality

I will work with the natural notions of compositionality concerning character and content that Dag Westerståhl and Peter Pagin (2010; 2012) have defined. In Predelli’s notation, which I will follow, $\lambda c.\lambda i. \llbracket \phi \rrbracket (c, i)$ is the *character* of an expression ϕ , and $\lambda i. \llbracket \phi \rrbracket (c, i)$ is the *content* of ϕ in a context c .¹ If the character of a complex expression is a function of those of its parts, we have what I will call **character-compositionality**. Thus, an operator M is character-compositional just in case there is a function f such that for every sentence ϕ : $\lambda c.\lambda i. \llbracket M\phi \rrbracket (c, i) = f(\lambda c.\lambda i. \llbracket \phi \rrbracket (c, i))$. If the content, in a given context, of a complex is a function of the contents, in that context, of its parts, we have what I will call **content-compositionality**. Thus, an operator M is content-compositional just in case there is a function f such that for every sentence ϕ ,

¹The one difference from Predelli’s notation is that for simplicity’s sake I collapse the world and time points of evaluation into one point i .

and every context c : $\lambda i. \llbracket M\phi \rrbracket(c, i) = f(\lambda i. \llbracket \phi \rrbracket(c, i))^2$

Westerståhl (2012, §5.2) proves that content-compositionality entails character-compositionality.

We will show that monsters in Predelli’s first category – the ‘context shifters’ – are character-compositional, by giving a format in which all the paradigmatic examples can be defined, and by showing that anything so definable is character-compositional.

3 Context shifters

3.1 Examples

The **context shifters** are, Predelli writes, ‘to be explained on the model of intensional operators, *modulo* the substitution of context sensitivity for circumstance sensitivity’ (390). Accordingly we can say that a context shifter is an operator the definition of which involves on the right-hand side the evaluation of the argument sentence with respect to some context or contexts other than the one specified on the left-hand side.³ This isn’t a precise definition of the context shifters, nor does Predelli pretend that it is; he rests with the above-quoted statement about what Kaplan’s idea ‘seems to be’ (390).

One way in which something worth calling ‘shifting’ of contexts can happen is that the right-hand side involves *quantification over* evaluations of the argument sentence in different contexts. Predelli gives one example of a context shifter, and it is of this sort.

For all c, i : $\llbracket \text{Never } \phi \rrbracket(c, i) = \top$ iff there is no context e , which differs from c at most by time, such that $\llbracket \phi \rrbracket(e, i) = \top$.

The ‘in some contexts’ example Kaplan (1977, 510) gives is (as Rabern notes, 402) definable in the same sort of way, this time by reference to C , the set of all contexts:

For all c, i : $\llbracket \text{In some contexts } \phi \rrbracket(c, i) = \top$ iff there is some context e , which is in C , such that $\llbracket \phi \rrbracket(e, i) = \top$.

Kaplan gives this example as part of the discussion that Predelli takes to introduce the very idea of context shifters; presumably it is a paradigm.

One might query, though, whether quantifying over contexts should be called ‘shifting’ them. This brings us to the second sort of way in which the right-hand side can involve evaluation at different contexts: we can evaluate at *one* context different from the one on the left-hand side. It could be a named context, or one

²Westerståhl (2012) calls the former ‘standard compositionality’ (§5.1) and the latter ‘contextual compositionality’ (§5.2). My formal statements here just cover the case of sentential operators, since that is the focus in almost all discussions of monsters. In general, of course, there can be monsters of many syntactic types, e.g. term-forming monsters.

³Rabern (2013) gives almost exactly this as his Definition 2 of monsters (395), which unlike his Definition 1 does not rest on an assumption about the compositionality of monsters.

specified as standing in a certain relation to the argument context. To illustrate the first possibility, let c' be some special context with respect to which one wants to define a monstrous operator. Then one can define an ‘In c' ’ monster that shifts evaluation to c' :

For all c, i : $\llbracket \text{In } c', \phi \rrbracket(c, i) = \top$ iff there is some context e , which is identical to c' , such that $\llbracket \phi \rrbracket(e, i) = \top$.

(To readers who wonder why I didn’t say ‘ c' is such that,’ instead of the more complex locution: as will become clear soon, I am highlighting commonalities among definitions of different context-shifting operators.) To illustrate the second possibility, one can define a ‘One day back’ operator that ‘shifts’ to the context the same except one day back:

For all c, i : $\llbracket \text{One day back } \phi \rrbracket(c, i) = \top$ iff there is some context e , which is the same as c except that its time is one day earlier, such that $\llbracket \phi \rrbracket(e, i) = \top$

There is clearly a ‘shift’ in both these cases, so they seem to merit the label ‘context shifters’ as much as (if not more than) the quantification-involving examples that Predelli and Kaplan give. As we will see, *all* these cases are the same in terms of compositionality, so it is not unreasonable to put them all under the same heading.

3.2 A general format for the context-shifters

In each case of a context-shifting operator M that we have discussed, the evaluation of $M\phi$ at (c, i) involves the evaluation of ϕ at a context or contexts other than c , the question being: *how many* of these are contexts that make ϕ true at i ? Thus, each operator gives a verdict concerning a pair of sets of contexts: the set of contexts that it is ‘interested in,’ and the set of contexts making the argument sentence true at i .

We can make these features of context-shifters explicit by saying two things. First, every operator M that we’ve discussed involves a selection function $Sel_M(c)$, which returns the set of contexts the operator is ‘interested in.’ (E.g. ‘In c' ’ is interested in context c' ; ‘In some contexts’ is interested in all contexts.) We need c as an argument to this function because in some cases those contexts are selected in virtue of their relation to c , as with ‘Never’ and ‘One day back.’ Second, every operator M that we’ve discussed returns a verdict on some relation between that set of contexts and the set of *all* contexts making the argument sentence true at the point of evaluation in question. (E.g. the ‘In c' ’ operator wants to know whether the one context it’s interested in is among those contexts; ‘Never’ wants to know whether *any* of the contexts it’s interested in is among those contexts.) We can formalize this as a comparison function $Comp_M$, taking as arguments a pair of sets of contexts, and returning a truth value.

We can now write a general format in which every operator so far discussed can be defined:

For all c, i : $\llbracket M\phi \rrbracket(c, i) = \text{Comp}_M(\text{Sel}_M(c), \{e : \llbracket \phi \rrbracket(e, i) = \top\})$.

With respect to the examples so far discussed, the relevant sets $\text{Sel}_M(c)$, and the functions Comp_M are as follows. (The column for Comp_M gives the condition for it to return \top .)

<i>Operator</i>	$\text{Sel}_M(c)$	$\text{Comp}_M(S, T)$
‘Never’	$\{d : d \text{ differs from } c \text{ at most by time}\}$	$S \cap T = \emptyset$
‘In some contexts’	C (the set of all contexts)	$S \cap T \neq \emptyset$
‘In c' ’	$\{c'\}$	$S \subseteq T$
‘One day back’	$\{d : d \text{ differs from } c \text{ only in that it is one day earlier}\}$	$S \subseteq T$

It is impressive that this format fits the examples of context shifters that Predelli and Kaplan give, the ones that I made up, and others besides.⁴ The similarity with generalized quantifiers is striking: in each case the comparison function concerns a simple set-theoretic relation. Since the first set in the comparison is a function of the argument context c , what we have could be described as a generalized quantifier whose arguments are that context and the set of contexts making the argument sentence true (at the point of evaluation at which the entire construction is being evaluated).⁵

Perhaps there are other operators that don’t fit this format, which some might want to call ‘context shifters.’ I will not argue over the label. Rather, I will argue that every operator that is definable along the same lines as the paradigmatic context shifters – that is, definable using our general format – is character-compositional.

3.3 Proving character-compositionality

3.3.1 Defining the function f involved in character-compositionality

To discuss character-compositionality we need to abstract on the context-point style of definition used above, to get something that explicitly equates characters:

$$\lambda c . \lambda i . \llbracket M\phi \rrbracket(c, i) = \lambda c . \lambda i . \text{Comp}_M(\text{Sel}_M(c), \{e : \llbracket \phi \rrbracket(e, i) = \top\}).$$

We can now show that any M so definable is character-compositional. For the sake of abbreviation, rather than writing ‘ $\lambda c . \lambda i . \llbracket \phi \rrbracket(c, i)$ ’ for the character of ϕ we will just write ‘ $\text{Char}(\phi)$.’ Then the second argument to Comp_M in the general format can be rewritten as $\{e : \text{Char}(\phi)(e)(i) = \top\}$. Substituting that into (the abstracted version of) our general format, we get:

$$\lambda c . \lambda i . \llbracket M\phi \rrbracket(c, i) = \lambda c . \lambda i . \text{Comp}_M(\text{Sel}_M(c), \{e : \text{Char}(\phi)(e)(i) = \top\}).$$

⁴I am grateful to Juhani Yli-Vakkuri for pressing me on the possibility of a monster involving ‘most’; in such a case $\text{Comp}_M(S, T)$ returns \top iff $|S \cap T| > |S - T|$.

⁵I am grateful to an anonymous referee for emphasizing this point.

Abstracting further, we can rewrite the right-hand side as the result of applying to $Char(\phi)$ a function taking characters as arguments:

$$\lambda c . \lambda i . \llbracket M\phi \rrbracket(c, i) = \left[\lambda K . \lambda c . \lambda i . Comp_M(Sel_M(c), \{e : K(e)(i) = \top\}) \right](Char(\phi)).$$

This tells us what the function f is, that the above definition of character-compositionality requires. It is:

$$\lambda K . \lambda c . \lambda i . Comp_M(Sel_M(c), \{e : K(e)(i) = \top\})$$

We have thus shown that any operator definable in our way is character-compositional: the character of $M\phi$ is the just-given function of the character of ϕ .

Character-compositionality is not a trivial feature; Yli-Vakkuri and Litland (2016) argue that the ‘definitely’ operator fails even to be character-compositional. More on that claim below.

3.3.2 A reassuring *reductio*

We can reassure ourselves of this result by arguing by *reductio* against the supposition that M is an operator definable in the general format we have given, $M\phi$ and $M\psi$ differ in character, and ϕ and ψ have the same character. Suppose that M is such an operator and that:

$$(1) \quad \lambda c . \lambda i . \llbracket M\phi \rrbracket(c, i) \neq \lambda c . \lambda i . \llbracket M\psi \rrbracket(c, i)$$

even though

$$(2) \quad \lambda c . \lambda i . \llbracket \phi \rrbracket(c, i) = \lambda c . \lambda i . \llbracket \psi \rrbracket(c, i)$$

By (1), there is some context d such that:

$$\lambda i . \llbracket M\phi \rrbracket(d, i) \neq \lambda i . \llbracket M\psi \rrbracket(d, i)$$

This in turn means that there is some point k such that:

$$\llbracket M\phi \rrbracket(d, k) \neq \llbracket M\psi \rrbracket(d, k)$$

Using our general format for the character of $M\phi$ and $M\psi$, this means:

$$\begin{aligned} & \left[\lambda c . \lambda i . Comp_M(Sel_M(c), \{e : \llbracket \phi \rrbracket(e, i) = \top\}) \right](d)(k) \\ \neq & \left[\lambda c . \lambda i . Comp_M(Sel_M(c), \{e : \llbracket \psi \rrbracket(e, i) = \top\}) \right](d)(k) \end{aligned}$$

Converting the lambda expressions, we get:

$$Comp_M(Sel_M(d), \{e : \llbracket \phi \rrbracket(e, k) = \top\}) \neq Comp_M(Sel_M(d), \{e : \llbracket \psi \rrbracket(e, k) = \top\})$$

This in turn entails:

$$\{e : \llbracket \phi \rrbracket(e, k) = \top\} \neq \{e : \llbracket \psi \rrbracket(e, k) = \top\}$$

But this clearly contradicts assumption (2), that ϕ and ψ have the same character. So, it is impossible for an operator M that is definable using our general format, applied to two sentences with the same character, to result in sentences with different characters. Any such operator is character-compositional.

3.4 Context-shifters are not content-compositional

What about the claim that context-shifters are not content-compositional? We cannot prove this by using our general format, since the identity operator too can be expressed in that format and it is content-compositional. (Recall that the format is not meant as a *definition* of ‘context shifter.’) But we can look at a couple of examples so far discussed, to show that context shifters are not, in general, content-compositional.

Consider the ‘In c' ’ operator, where c' is a context in which Donald Trump is the speaker and the location is the White House. Let c^E be some context in which Albert Einstein is the speaker and the location is the Institute for Advanced Studies. Then ‘Einstein is at the IAS’ and ‘I am here’ have the same content when taken in c^E . Now consider what the operator does to these sentences.⁶ (Recall from the table that the function $Comp_M$ for the operator ‘In c' ’ returns \top iff the first argument to it is a subset of the second.)

$$\begin{aligned} & \llbracket \text{In } c' \text{ Einstein is at the IAS} \rrbracket (c^E, i) = \top \\ \text{iff } & \text{Comp}_M(\{c'\}, \{e : \llbracket \text{Einstein is at the IAS} \rrbracket (e, i) = \top\}) = \top \\ \text{iff } & \{c'\} \subseteq \{e : \llbracket \text{Einstein is at the IAS} \rrbracket (e, i) = \top\} \\ \text{iff } & \llbracket \text{Einstein is at the IAS} \rrbracket (c', i) = \top \\ \text{iff } & \text{Einstein is at the IAS, at } i \end{aligned}$$

while

$$\begin{aligned} & \llbracket \text{In } c' \text{ I am here} \rrbracket (c^E, i) = \top \\ \text{iff } & \text{Comp}_M(\{c'\}, \{e : \llbracket \text{I am here} \rrbracket (e, i) = \top\}) = \top \\ \text{iff } & \{c'\} \subseteq \{e : \llbracket \text{I am here} \rrbracket (e, i) = \top\} \\ \text{iff } & \llbracket \text{I am here} \rrbracket (c', i) = \top \\ \text{iff } & \text{Trump is at the White House, at } i \end{aligned}$$

Thus, *different* contents (in c^E) result from the application of the ‘In c' ’ operator to sentences having the same content (in c^E). ‘In c' ’ is not content-compositional.

What about ‘In some contexts’? Consider ‘I am not here,’ with context c' as above. Taken in c' , that sentence has the same content as ‘Donald Trump is not at the White House.’ Yet the contents, when taken in c' , of ‘In some contexts Trump is not at the White House’ and ‘In some contexts, I am not here’ differ.⁷ ‘In some contexts’ is not content-compositional.

⁶In these calculations I ignore the role of tense in the argument sentence.

⁷Recall that for this operator, $Comp_M$ returns \top iff $S \cap T \neq \emptyset$. Then:

$$\begin{aligned} & \llbracket \text{In some contexts Trump is not at the White House} \rrbracket (c', i) = \top \\ \text{iff } & \text{Comp}_M(C, \{e : \llbracket \text{Trump is not at the White House} \rrbracket (e, i) = \top\}) = \top \\ \text{iff } & C \cap \{e : \llbracket \text{Trump is not at the White House} \rrbracket (e, i) = \top\} \neq \emptyset \\ \text{iff } & \{e : \llbracket \text{Trump is not at the White House} \rrbracket (e, i) = \top\} \neq \emptyset \\ \text{iff } & \text{For some } e : \llbracket \text{Trump is not at the White House} \rrbracket (e, i) = \top \\ \text{iff } & \text{Trump is not at the White House, at } i \end{aligned}$$

So we have shown that the paradigmatic context-shifters – all of which are definable in the general format – are character-compositional but not, in general, content-compositional.

4 Global shifters

Global shifters shift *context-relativized truth* (Kaplan 1977, 522) rather than context. Corresponding to the context shifter ‘In some contexts’ as defined above, for example, we would have, Predelli says, the global shifter ‘In some contexts^G’:

$$\llbracket \text{In some contexts}^G \phi \rrbracket(c, i_c) = \top \quad \text{iff} \quad \exists d : \llbracket \phi \rrbracket(d, i_d) = \top$$

(Here i_d is the circumstance ‘of’ the context d .)

The first thing to say about this is that it is not a full specification. It tells us only the value at context-circumstance pairs of a special kind: those in which the circumstance is the circumstance of the context. The would-be definition is silent on what the values are for other context-circumstance pairs. Perhaps we should think of global shifters as each being a *family* of operators that satisfy the stated condition. But that is an awkward thing to have to say, since it means that our English locution does not, in fact, capture the meaning of any one global shifter.

Since it’s not clear that there is any notion helping us to fully define any particular operator of the alleged kind, I will say no more about the category of global shifters; their very definability is dubious. My main point concerns the relation between context shifters and the third kind of monster Predelli discusses.

5 Character shifters

Predelli claims that there is a third kind of monster, the **character shifters** – ‘operators on character’ (392). He doesn’t give a formally defined example, perhaps because (following Kaplan’s (1977, 511) suggestion) he maintains that they are ‘unobjectionably expressible in English only with the appeal of pure quotation’ and the Logic of Demonstratives contains no quotation constructions.

which is false for many i , while

$$\begin{aligned} & \llbracket \text{In some contexts I am not here} \rrbracket(c', i) = \top \\ \text{iff} & \quad \text{Comp}_M(C, \{e : \llbracket \text{I am not here} \rrbracket(e, i) = \top\}) = \top \\ \text{iff} & \quad C \cap \{e : \llbracket \text{I am not here} \rrbracket(e, i) = \top\} \neq \emptyset \\ \text{iff} & \quad \{e : \llbracket \text{I am not here} \rrbracket(e, i) = \top\} \neq \emptyset \\ \text{iff} & \quad \text{For some } e : \llbracket \text{I am not here} \rrbracket(e, i) = \top \\ \text{iff} & \quad \text{For some } e : e_A \text{ is not at } e_P, \text{ at } i \end{aligned}$$

which is guaranteed to be the case, on reasonable assumptions about the variety of contexts and circumstances of evaluation.

What is it to ‘operate on character’? Presumably the idea is that it is to take a character as an input, and deliver a character as an output. It is difficult to see what notion other than this functional one could reasonably be attached to the phrase. But we have already seen this notion: it is character-compositionality, defined in explicitly functional terms. And we have already seen that the paradigmatic examples of context shifters, and anything similarly definable, *are* operators on character, since they are character-compositional. (This is the only claim we can make about what they operate on, since they are not content-compositional.) So the idea of a category of monsters that operate on character is the idea of a category that *includes* all the context shifters we know of. This seems to conflict with Predelli’s introduction of the categories as ‘non-equivalent’ (389).

To take stock: so far we have no reason to take there to be *any* monster that is not an ‘operator on character.’ The context shifters we know of operate on character (and not on content), and no example of a global shifter has even been described.

But what of the idea that character shifters necessarily involve quotation? That conflicts with the claims just summarized, since – as we have seen – the context shifting operators we know of are definable without the use of quotation.

The idea of an operator on character actually suggests the absence, rather than the presence, of quotationality. There can be distinct sentences with the same character. In such a case an ‘operator on character’ would produce the *same* output for the two sentences. This is no reason to think that in its calculation such an operator nonetheless takes account of *distinct* items (namely, the quotations of the distinct sentences). On the contrary, it is an operator’s *failure* to be even character-compositional that should make us ask whether it is covertly quotational. (This is one claim worth considering in relation to the above-cited claim of Yli-Vakkuri and Litland concerning the ‘definitely’ operator.) For such a failure would consist in there being cases in which the operator produces different results when applied to sentences with the same character. An explanation of this would have to advert to some difference between the sentences – hence, to some feature of theirs other than the character (and content) that they share. One obvious candidate would be their being different sentences, which is a difference expressible using quotation.⁸

⁸One other possible consideration (which I have no reason to attribute to anyone) goes as follows. With the use of quotation we can form character-denoting expressions, e.g. ‘the character of “Ann smokes”.’ Thus we could make explicit an operator’s taking characters as arguments by thinking of it as taking such quotation-involving expressions as arguments. So the thought might go. The problem with it is that we *could* do that with operators on contents as well, seeing their arguments as best expressible as ‘the content of “Ann smokes”’ and the like. So the possibility of such a construal in the character-operator case isn’t a reason to think of it as covertly quotational.

6 Conclusion

Our examination of Predelli’s three categories gives us no reason to think that there are any monsters not in the third category. The context shifters that we know of *are* operators on character and not on content,⁹ and it is doubtful that the global shifters are even definable. What this suggests is that being a context shifter is simply one way of being an operator on character. The fact that context shifters don’t involve quotation isn’t a problem for this claim, since there seems to be nothing to recommend the idea that operators on character must do so.¹⁰

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⁹While I quoted Rabern making this claim, in other parts of his (2013) he speaks of ‘compositionality’ without always stating which sort he means. His insistence (e.g. in his Definition 1, mentioned above, fn. 4) on the compositionality of monsters has the consequence, he claims (402), that ‘In some contexts’ (as defined above) isn’t a monster. (This is an awkward consequence, since it is the single example of a monster that Kaplan gives.) Since that operator is character-compositional but not content-compositional, its exclusion by the compositionality requirement must mean that what it requires is *content-compositionality*.

¹⁰For comments on previous versions of this paper I am grateful to Philip Kremer and Juhani Yli-Vakkuri.

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Yli-Vakkuri, Juhani, and Jon Litland. 2016. Vagueness and modality. *Philosophical Perspectives* 30:229–69.